C & EE 141

Bending Members

Laterally Unsupported and Noncompact

Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

- *Only limited to these three limit states when compression flange is laterally braced and beam is compact.*

What if the compression flange isn't braced? What if there is a local buckling issue?

Example of an Unbraced Beam

Limit States for Flexure

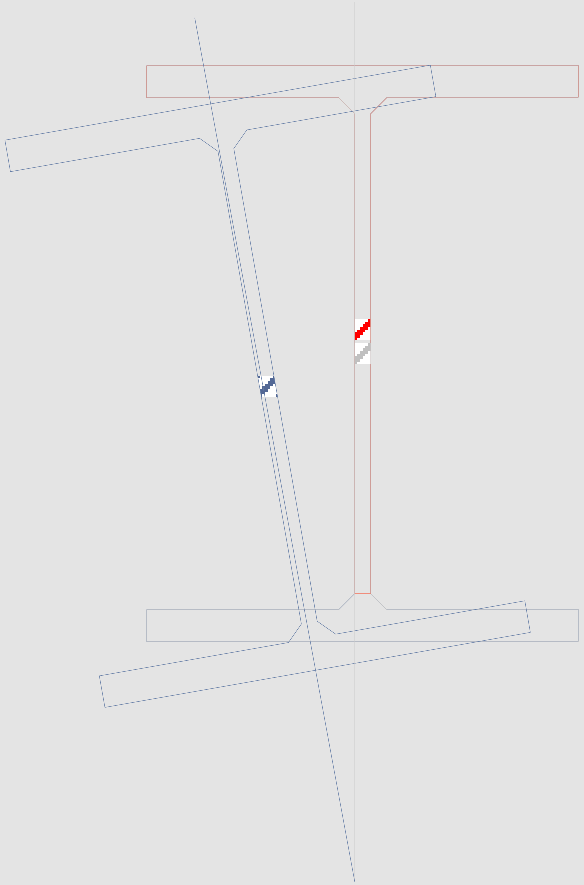
- Plastic Flexural Capacity
- **Global Buckling**
  - **Inelastic lateral torsional buckling**
  - **Elastic lateral torsional buckling**
- **Local Buckling**
  - **Width-thickness ratios for web and**
- Shear Capacity
- Deflection

Limit States for Flexure

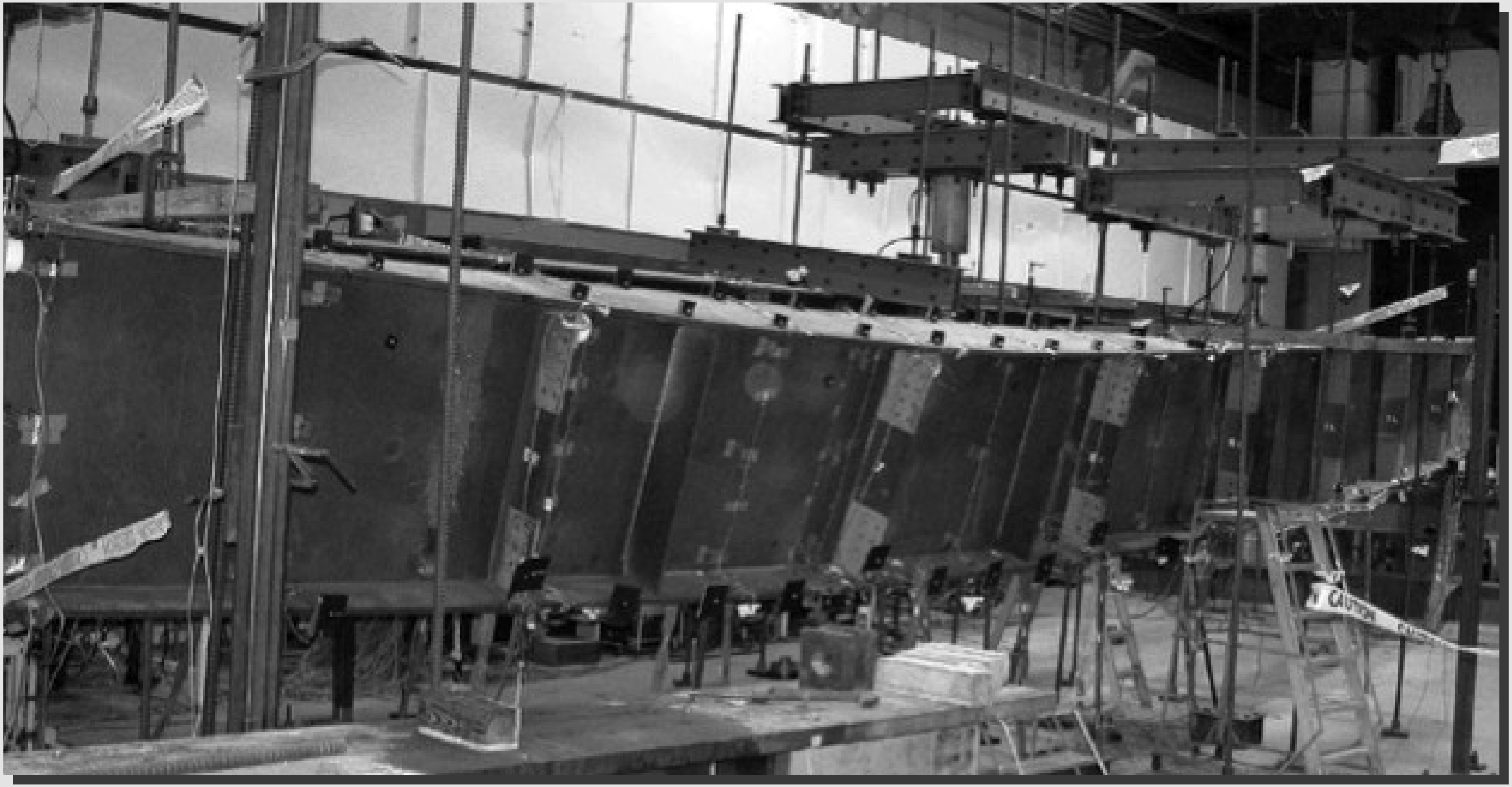
- Plastic Flexural Capacity
- Global Buckling
  - Inelastic lateral torsional buckling
  - Elastic lateral torsional buckling
- Local Buckling
  - Width-thickness ratios for web and flanges
- Shear Capacity
- Deflection

What is Lateral Torsional Buckling?

- The compression zone of the section buckles if not restrained
- For a simply supported WF that is the top flange and, to a varying extent, the web
- For a cantilevered WF that is the bottom flange and, to a varying extent, the web



What is Lateral Torsional Buckling?



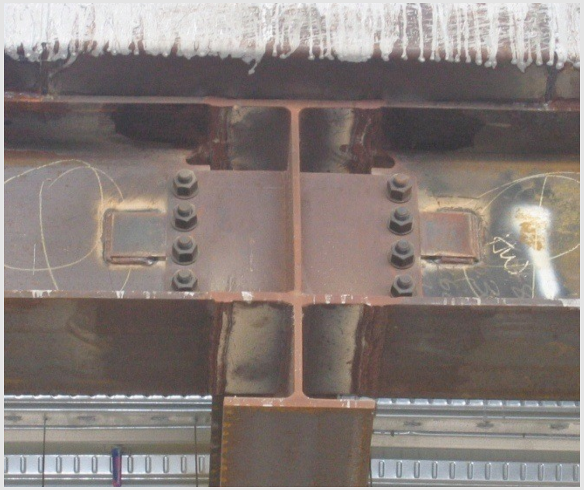
Onset of Lateral Torsional Buckling

Elastic LTB

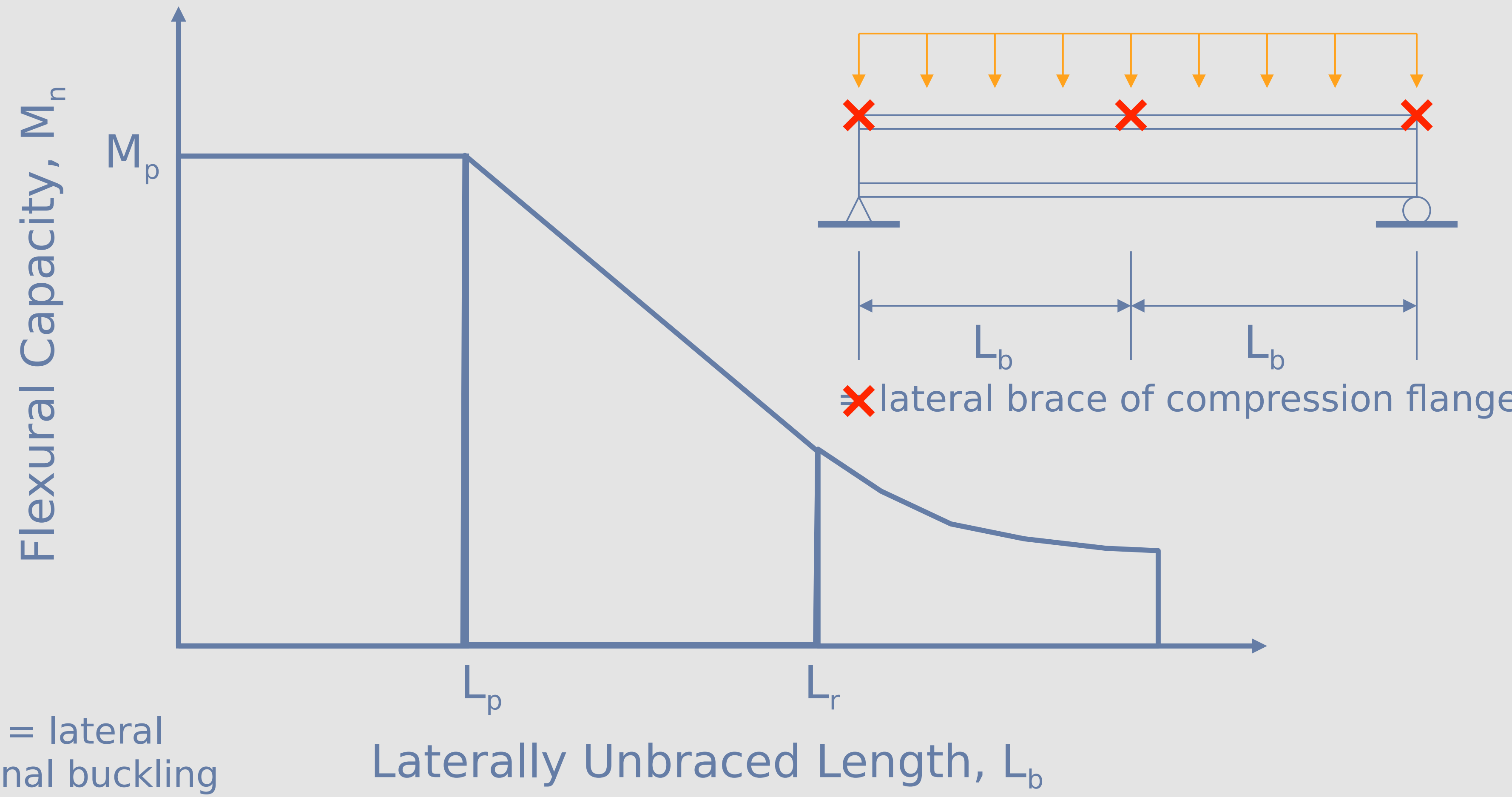
How is Lateral Torsional Buckling Prevented?

- Headed steel studs welded to the beam flange harden in the concrete fill which provides continuous stability

- Connections from perpendicular framing beams provide discrete points of stability

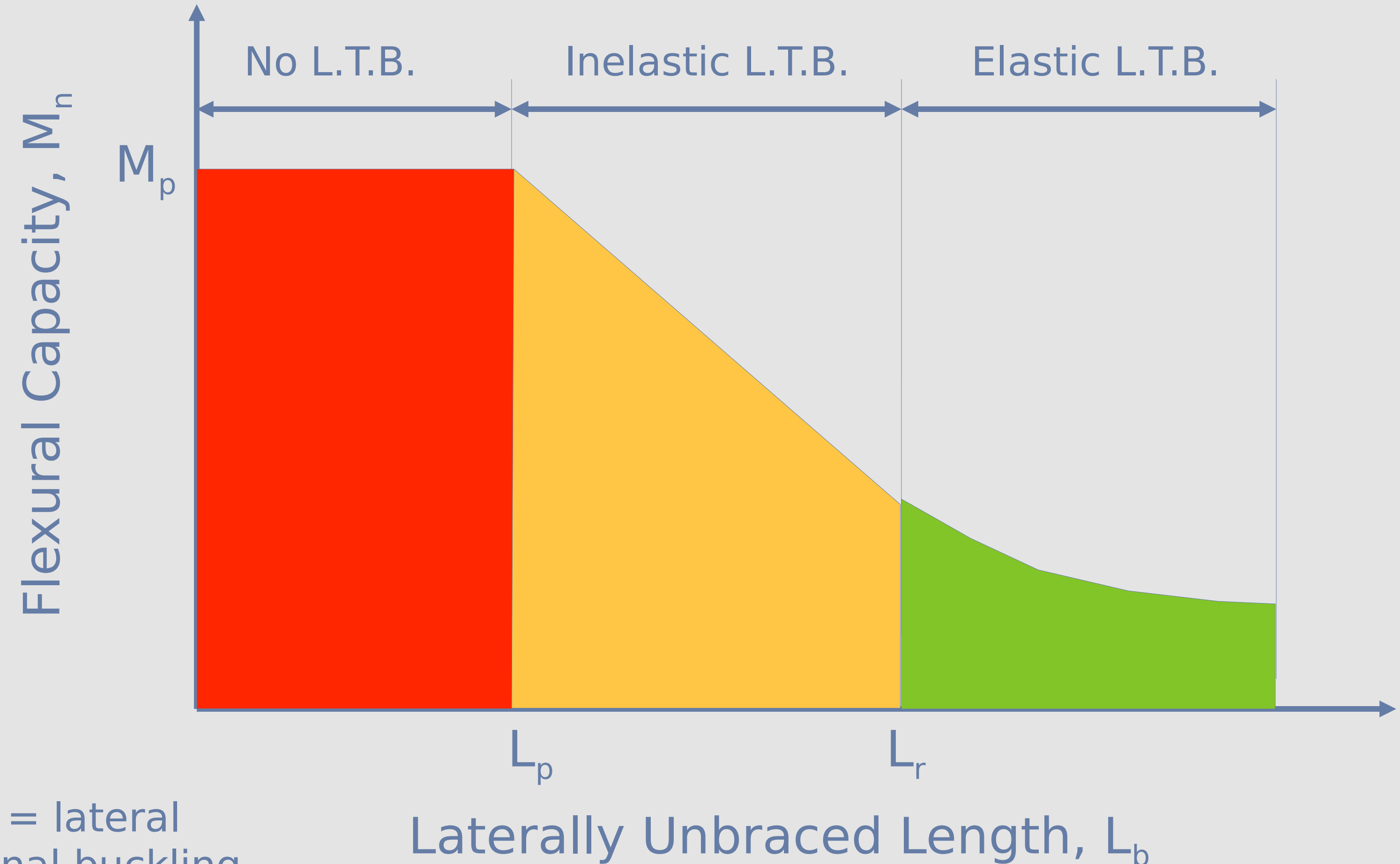


Transitions in Flexural Capacity due to Global Buckling



L.T.B. = lateral torsional buckling

Transitions in Flexural Capacity due to Global Buckling



L.T.B. = lateral torsional buckling

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric I-shaped members and channels bent about their major axis, having compact webs and compact flanges as defined in Section B4.1 for flexure.

> **User Note:** All current ASTM A6 W, S, M, C and MC shapes except W21×48, W14×99, W14×90, W12×65, W10×12, W8×31, W8×10, W6×15, W6×9, W6×8.5 and M4×6 have compact flanges for F_y = 50 ksi (345 MPa); all current ASTM A6 W, S, M, HP, C and MC shapes have compact webs at $F_y \leq 65$ ksi (450 MPa).

The *nominal flexural strength*, M_n, shall be the lower value obtained according to the *limit states* of *yielding (plastic moment)* and *lateral-torsional buckling*.

1. Yielding

$$M_n = M_p = F_y Z_x \qquad \text{(F2-1)}$$

where

F_y = *specified minimum yield stress* of the type of steel being used, ksi (MPa)

Z_x = plastic section modulus about the x-axis, in.^{3} (mm^3)

2. Lateral-Torsional Buckling

(a) When $L_b \le L_p$, the *limit state* of *lateral-torsional buckling* does not apply.

(b) When $L_p < L_b \le L_r$

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(F2-2)}$$

(c) When $L_b > L_r$

$$M_n = F_{cr}S_x \le M_p \qquad \text{(F2-3)}$$

where

L_b = length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section, in. (mm)

$$F_{cr} = \frac{C_b\pi^2E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1+0.078\frac{Jc}{S_xh_o}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(F2-4)}$$

and where

E = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)

J = torsional constant, in.^{4} (mm^4)

S_x = elastic section modulus taken about the x-axis, in.^{3} (mm^3)

h_o = distance between the flange centroids, in. (mm)

The limiting lengths L_p and L_r are determined as follows:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \tag{F2-5}$$

$$\boxed{L_r = 1.95 r_{ts} \frac{E}{0.7 F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76 \left(\frac{0.7 F_y}{E}\right)^2}}} \tag{F2-6}$$

where

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \tag{F2-7}$$

and the coefficient c is determined as follows:

(a) For doubly symmetric I-shapes: $c = 1$ (F2-8a)

(b) For channels: $c = \frac{h_o}{2} \sqrt{\frac{I_y}{C_w}}$ (F2-8b)

User Note: The square root term in Equation F2-4 may be conservatively taken equal to 1.0.

User Note: Equations F2-3 and F2-4 provide identical solutions to the following expression for lateral-torsional buckling of doubly symmetric sections that has been presented in past editions of the AISC LRFD Specification:

$$M_{cr} = C_b \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$

The advantage of Equations F2-3 and F2-4 is that the form is very similar to the expression for lateral-torsional buckling of singly symmetric sections given in Equations F4-4 and F4-5.

User Note: For doubly symmetric I-shapes with rectangular flanges, $C_w = \frac{I_y h_o^2}{4}$ and thus Equation F2-7 becomes

$$r_{ts}^2 = \frac{I_y h_o}{2S_x}$$

r_{ts} may be approximated accurately and conservatively as the radius of gyration of the compression flange plus one-sixth of the web:

$$r_{ts} = \frac{b_f}{\sqrt{12\left(1 + \frac{1}{6}\frac{ht_w}{b_f t_f}\right)}}$$

Global Buckling Limits

1. Inelastic LTB ($L_p < L_b \le L_r$)

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

(Eq F2-2)

$$\phi_b M_n = C_b\left[\phi_b M_p - \phi_b BF\left(L_b - L_p\right)\right] \le \phi_b M_p$$

(pg 3-9)

Table 3-2

2. Elastic LTB ($L_b > L_r$)

$$M_n = F_{cr}S_x \le M_p$$

(Eq F2-3) where F_{cr} per Eq F2-4

F_y = 50 ksi

Table 3-2 (continued)
W-Shapes
Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W30×116	**378**	**943**	**1420**	**575**	**864**	**24.8**	**37.4**	**7.74**	**22.6**	**4930**	**339**	**509**
W21×147	373	931	1400	575	864	13.7	20.7	10.4	36.3	3630	318	477
W24×131	370	923	1390	575	864	16.3	24.6	10.5	31.9	4020	296	445
W18×158	356	888	1340	541	814	10.5	15.9	9.68	42.8	3060	319	479
W14×193	355	886	1330	541	814	5.30	7.93	14.3	79.4	2400	276	414
W12×210	348	868	1310	510	767	4.25	6.45	11.6	95.8	2140	347	520
W30×108	**346**	**863**	**1300**	**522**	**785**	**23.5**	**35.5**	**7.59**	**22.1**	**4470**	**325**	**487**
W27×114	343	856	1290	522	785	21.7	32.8	7.70	23.1	4080	311	467
W21×132	333	831	1250	515	774	13.2	19.9	10.3	34.2	3220	283	425
W24×117	327	816	1230	508	764	15.4	23.3	10.4	30.4	3540	267	401
W18×143	322	803	1210	493	740	10.3	15.7	9.61	39.6	2750	285	427
W14×176	320	798	1200	491	738	5.20	7.83	14.2	73.2	2140	252	378

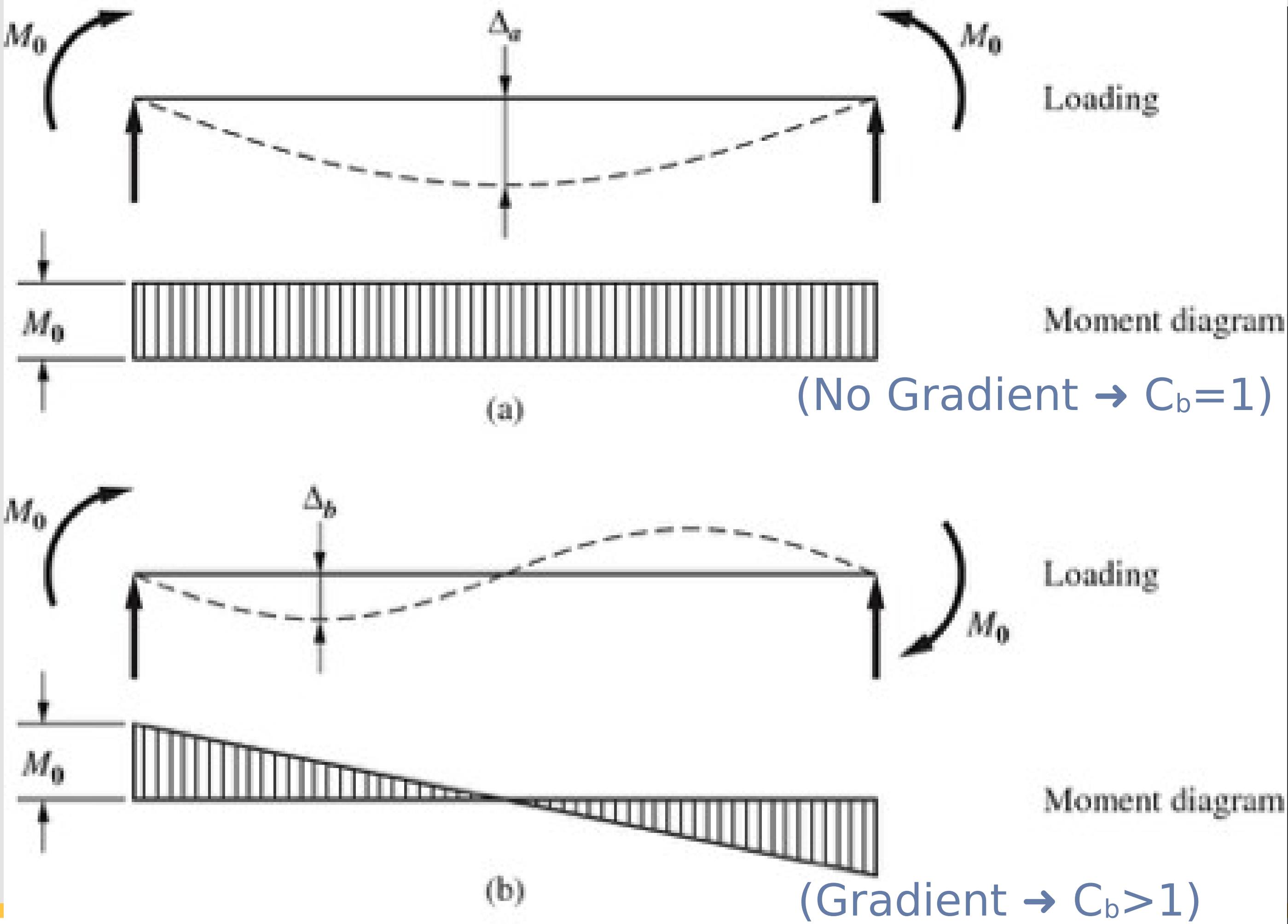
ASD	LRFD
$\Omega_b = 1.67$ $\Omega_v = 1.50$	$\phi_b = 0.90$ $\phi_v = 1.00$

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 50 ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

The Moment Gradient (C_b)

- C_b is a coefficient used to account for the effect of "moment gradients" on LTB
- Load pattern and end restraints change the moment diagram of the beam so that the effective length of the "compression" element of a beam may change
- C_b is needed because all calculations and tables are based on a case with

The Moment Gradient (C_b)



(No Gradient → C_b=1)

(Gradient → C_b>1)

The Moment Gradient (C_b)

(3) For singly symmetric members in *single curvature* and all doubly symmetric members:

C_b, the *lateral-torsional buckling* modification factor for nonuniform moment diagrams when both ends of the segment are braced is determined as follows:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad \text{(F1-1)}$$

where

M_{max} = absolute value of maximum moment in the unbraced segment, kip-in. (N-mm)

M_A = absolute value of moment at quarter point of the unbraced segment, kip-in. (N-mm)

M_B = absolute value of moment at centerline of the unbraced segment, kip-in. (N-mm)

M_C = absolute value of moment at three-quarter point of the unbraced segment, kip-in. (N-mm)

For cantilevers or overhangs where the free end is unbraced, C_b = 1.0.

The Moment Gradient (C_b)

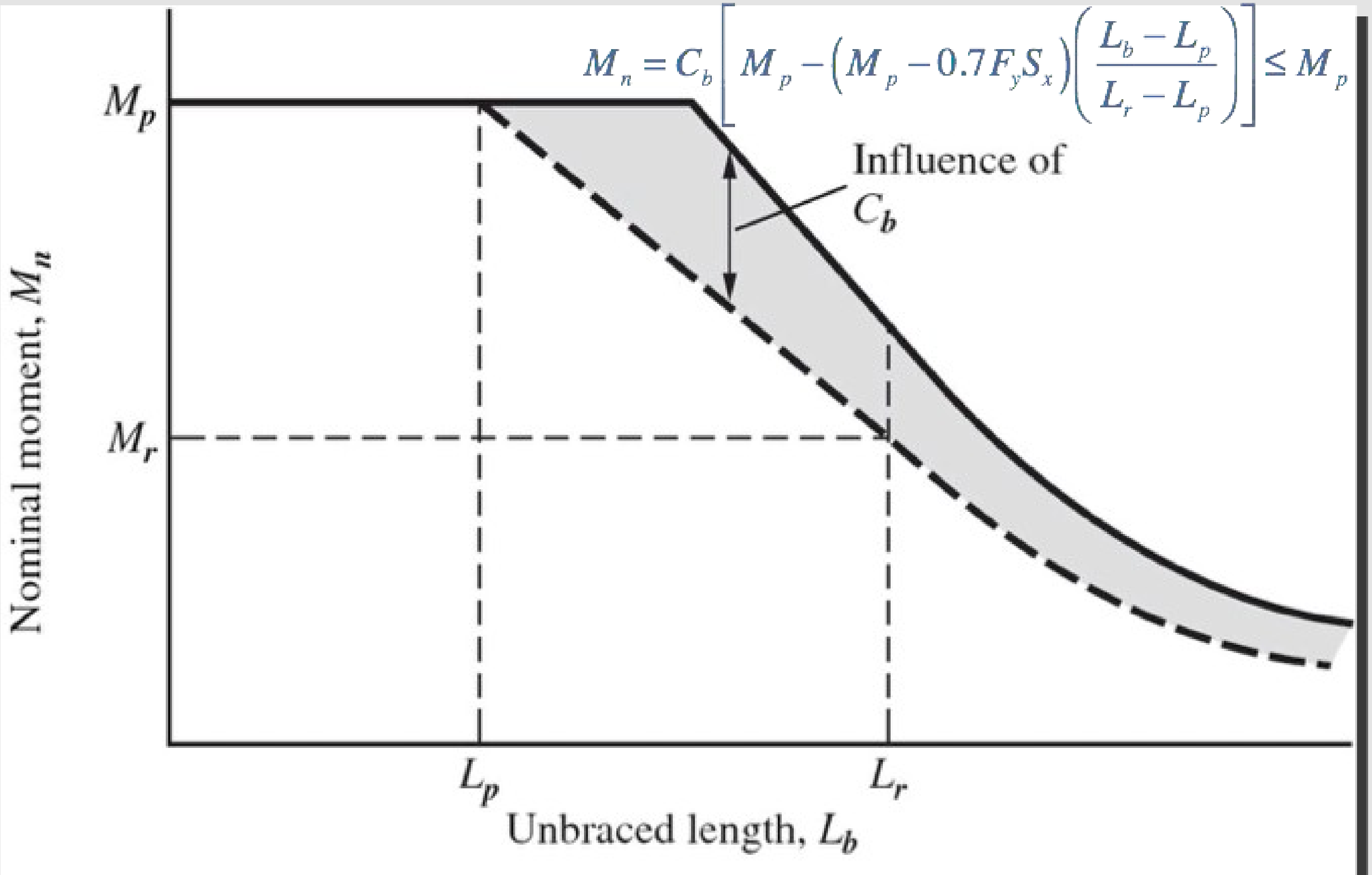
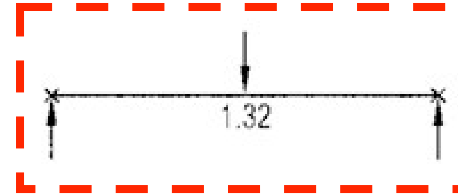


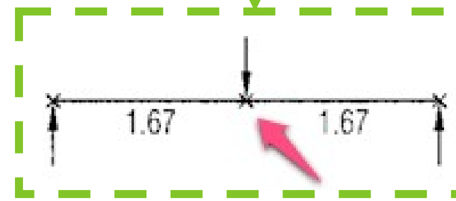
Figure 6.14

Table 3-1
Values of C_b for Simply Supported Beams

Load	Lateral Bracing Along Span	C_b
P	None Load at midpoint	1.32
	At load point	1.67 1.67
P P	None Loads at third points	1.14
	At load points Loads symmetrically placed	1.67 1.00 1.67
P P P	None Loads at quarter points	1.14
	At load points Loads at quarter points	1.67 1.11 1.11 1.67
w	None	1.14
	At midpoint	1.30 1.30
	At third points	1.45 1.01 1.45
	At quarter points	1.52 1.06 1.06 1.52
	At fifth points	1.56 1.12 1.00 1.12 1.56

Note: Lateral bracing must always be provided at points of support per AISC *Specification* Chapter F.





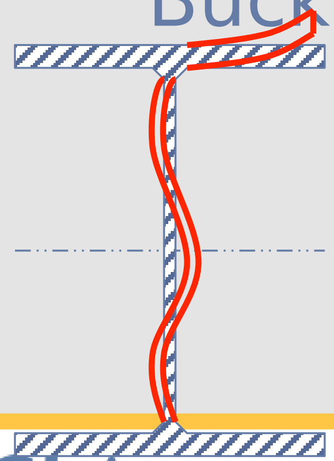
The little "x" indicates a lateral bracing point

Limit States for Flexure

- Plastic Flexural Capacity
- Global Buckling
  - Inelastic lateral torsional buckling
  - Elastic lateral torsional buckling
- Local Buckling
  - Width-thickness ratios for web and flanges
- Shear Capacity
- Deflection

Local Instability

- Buckling of one of the compression elements of the x–section.
  - Flange Local Buckling
  - Web Local Buckling



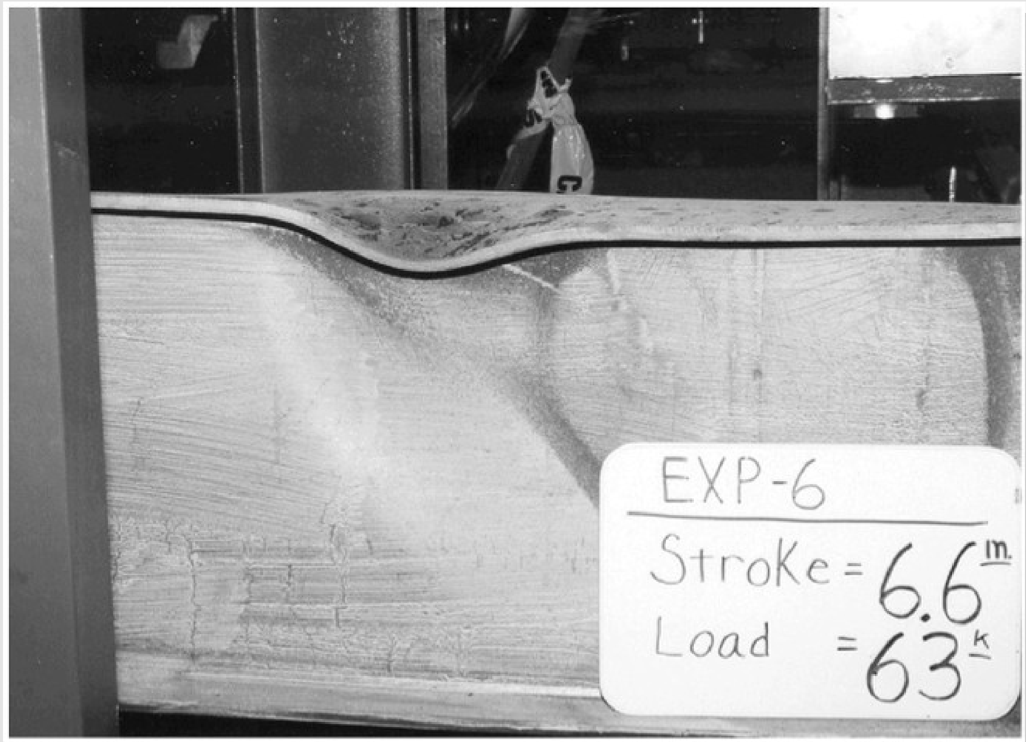
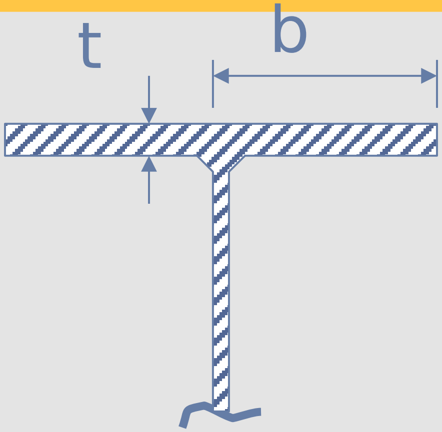


Figure 6.16
Photo courtesy of Donald W. White

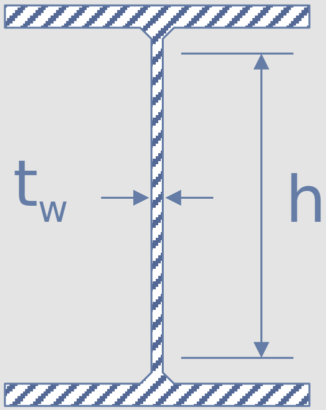
Local Instability

- Whether a member is subject to a local instability is a function of the width-thickness ratio of the elements that can buckle:



  - Flange Local Buckling

$$\lambda = b/t$$



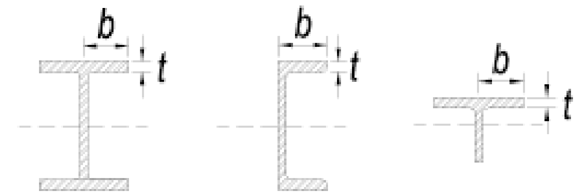
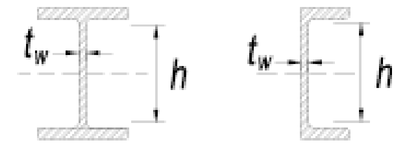
  - Web Local Buckling

$$\lambda = h/t_w$$

Checking the Ratios

- Limiting width-thickness ratios are function of:
  - geometry of the shapes
  - yield stress of the steel
- Shapes can be classified as:
  - A **compact shape** is not subject to local buckling.
  - A **non-compact shape** is subject to inelastic local buckling after initial yielding.
  - A **slender shape** is subject to elastic L.B.
- Local buckling will limit the moment capacity that can be reached by the

Classification of Sections for Local Buckling

Case	Description of Element	(λ) Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_p (compact/noncompact)	Limiting Width-to-Thickness Ratio λ_r (noncompact/slender)	Examples
10	Flanges of rolled I-shaped sections, channels, and tees	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
15	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	

Spec Table B4.1b

Effect on Flexural Capacity due to Local Buckling

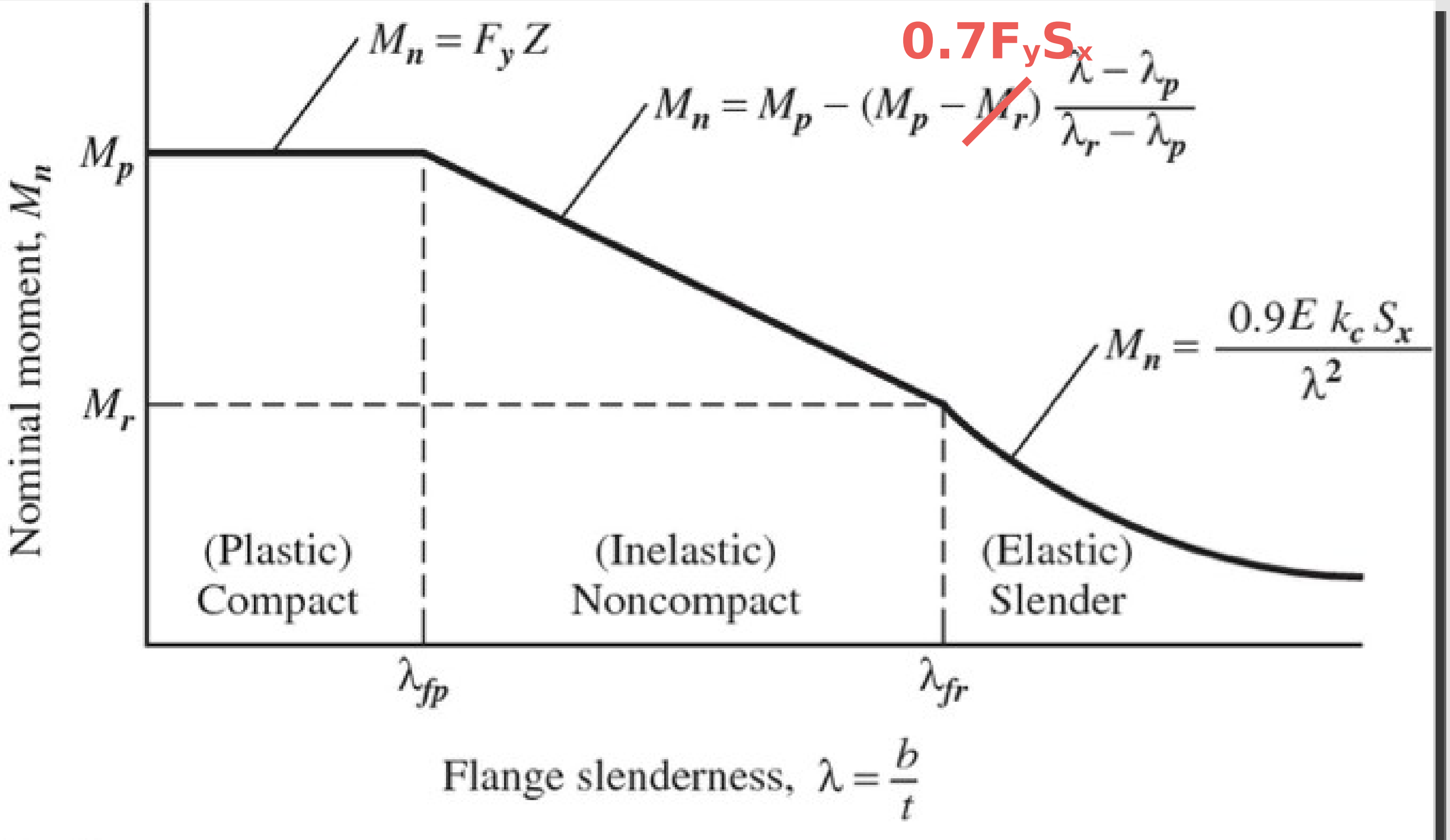


Figure 6.18

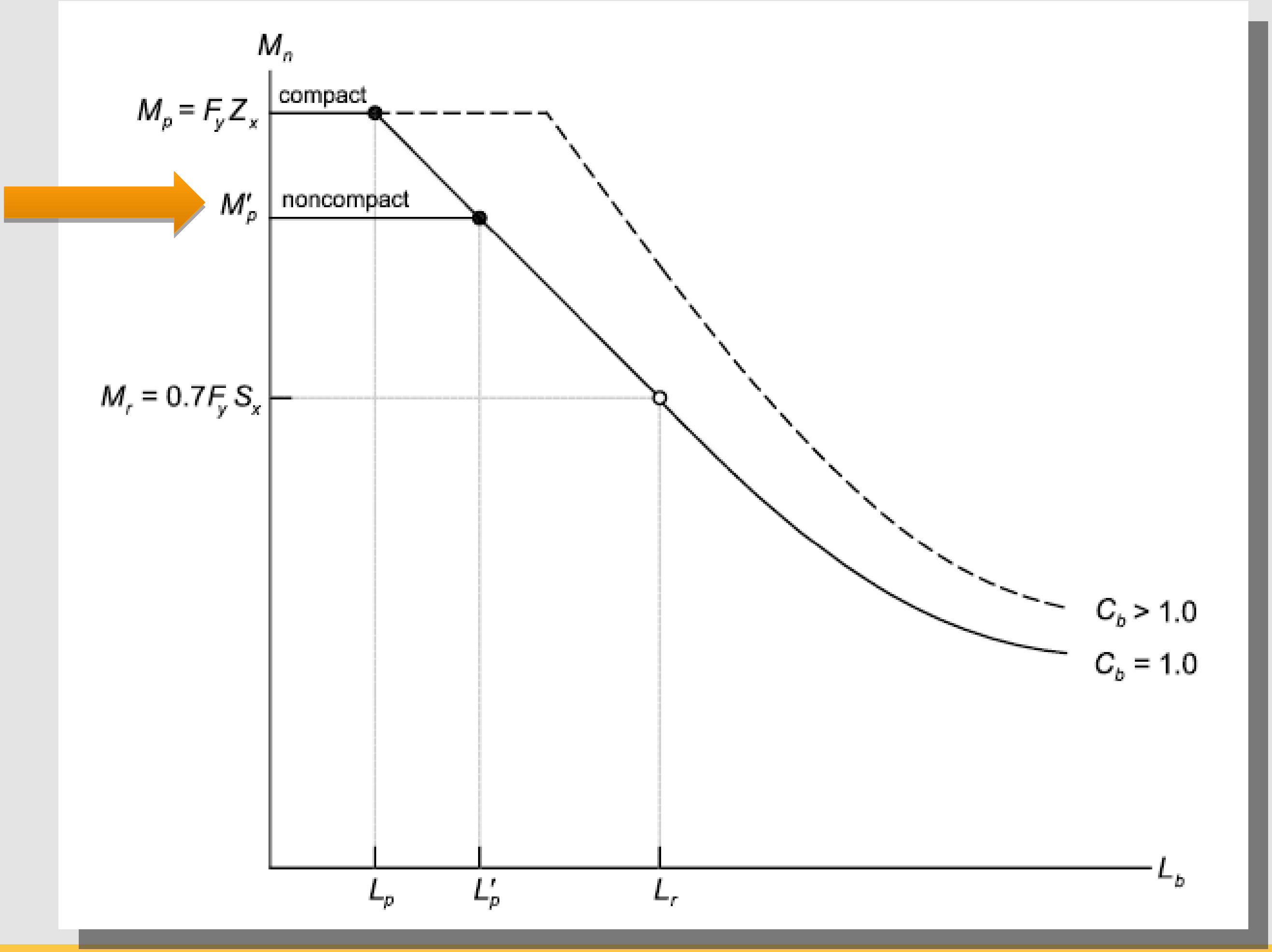
Maximum Flexural Capacity for Noncompact Sections

- Maximum moment capacity limited, regardless of LTB capacity:
  - For WF beam with noncompact flanges:

$$M_n = M_p - \left(M_p - 0.7F_yS_x\right)\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}$$

  - WF beams with slender flanges, or noncompact/slender webs trigger additional checks. (Not covered in this class).

Effect of Noncompact Section on Moment Capacity



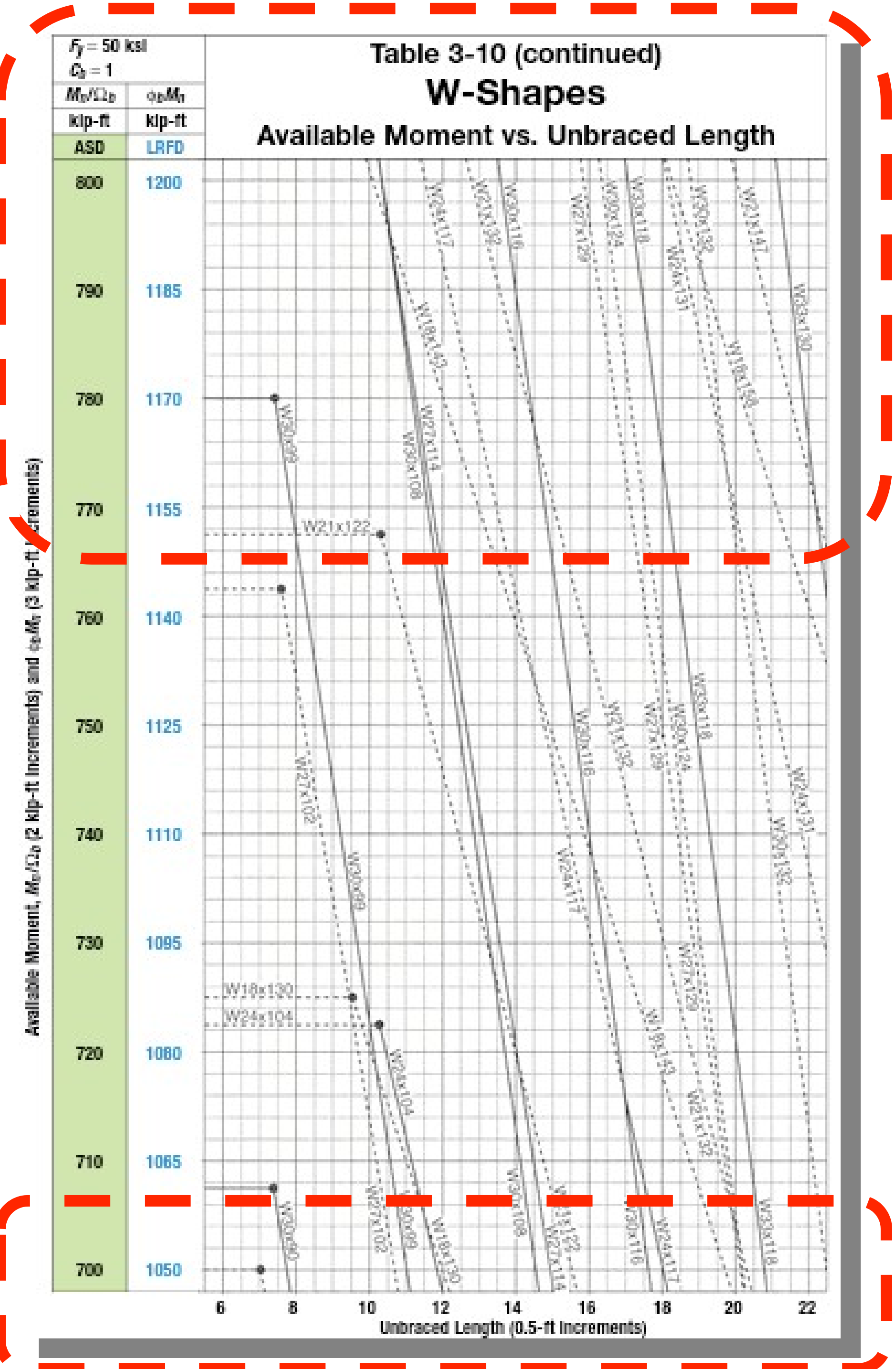
Design Checks for Flexure

1. Lateral Torsional Buckling
   - Checked for adequate lateral support of the compression flange.
2. Flange Local Buckling
   - Checked with $b_f/2t_f$ ratio.
3. Web Local Buckling
   - Checked with h/t_w ratio.
4. Formation of Plastic Moment
   - $M_u \leq \phi_b M_p$

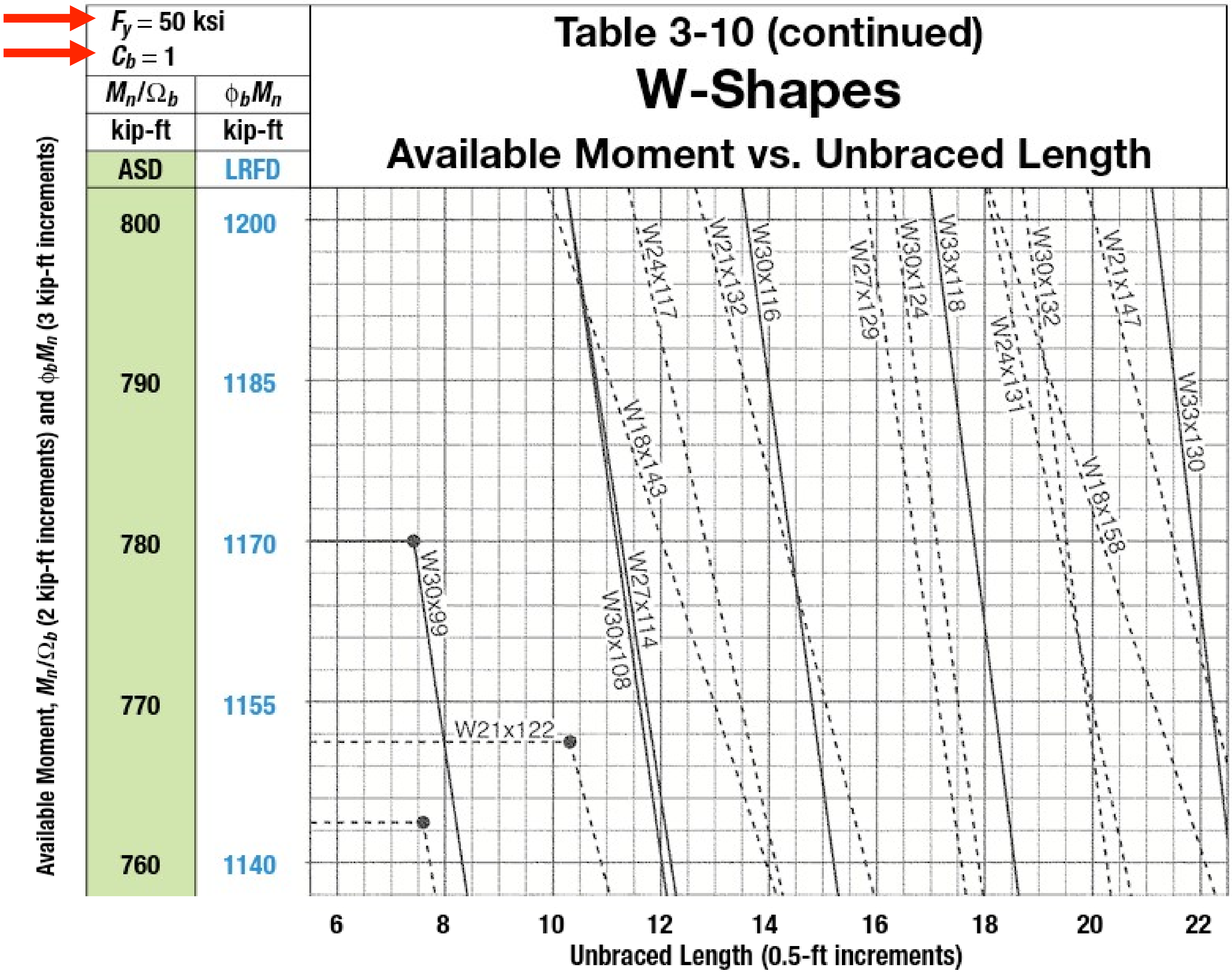
Do We Need All These Equations?

- Refer to AISC
  - Section Properties
    - Table 1-1: width-thickness ratios, S, I, Z
  - Beam Tables
    - Table 3-2: select most economical size based on Z_x
    - Table 3-3: select most economical size based on I_x
    - Table 3-10: select most economical size for particular unbraced length
      - Note: compactness of section is considered in tables

Beam Tables Based on Strong Axis Strength & Unbraced Length

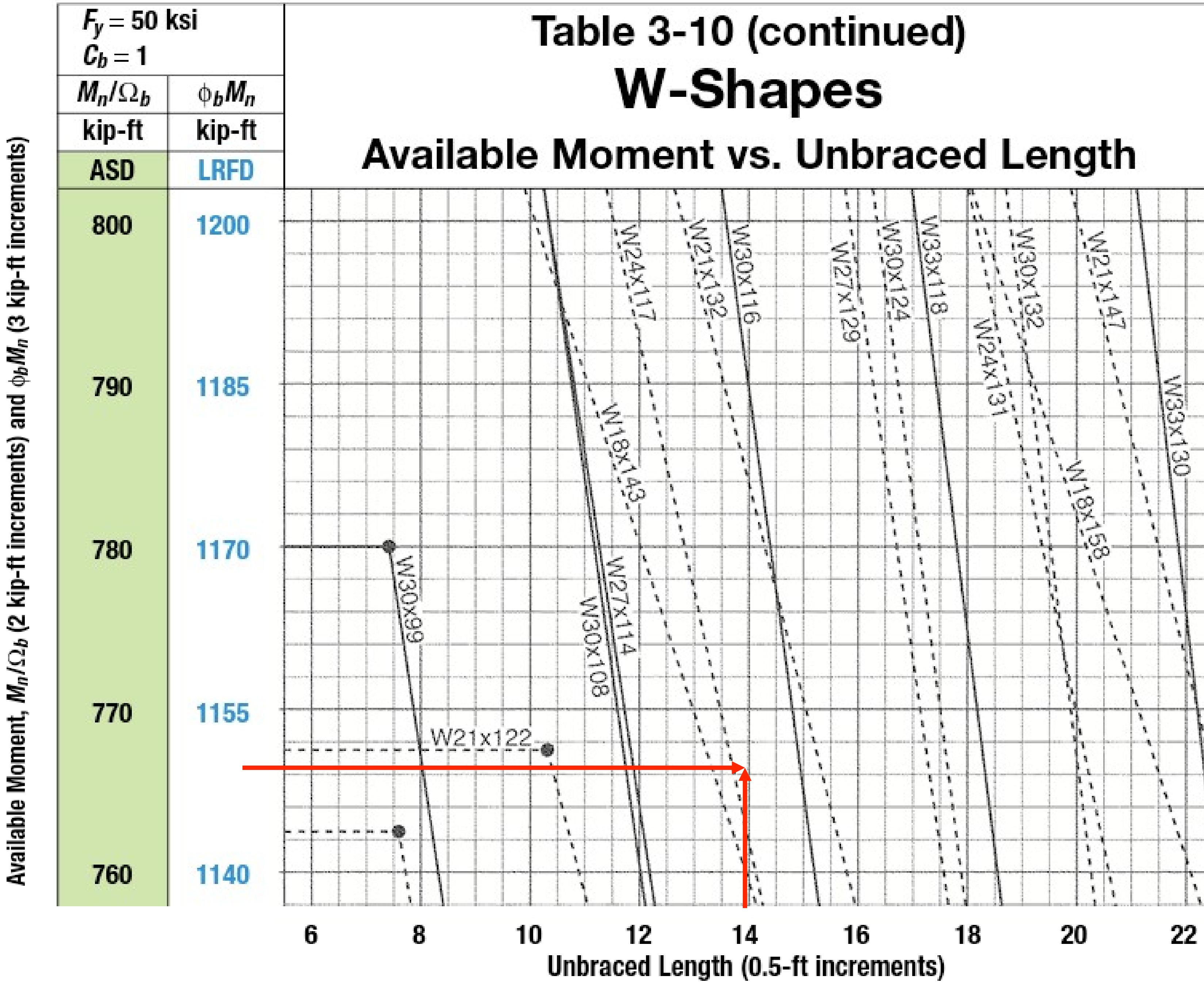


- Actual unbraced length (L_b) of the beam is accounted for
- Very commonly used table



$F_y = 50$ ksi
$C_b = 1$
Table 3-10 (continued)
W-Shapes
Available Moment vs. Unbraced Length
M_n/Ω_b
$\phi_b M_n$
kip-ft
kip-ft
ASD
LRFD
800
1200
790
1185
780
1170
770
1155
760
1140
Available Moment, M_n/Ω_b (2 kip-ft increments) and $\phi_b M_n$ (3 kip-ft increments)
6
8
10
12
14
16
18
20
22
Unbraced Length (0.5-ft increments)
W24x117
W21x132
W30x116
W27x129
W30x124
W33x118
W24x131
W30x132
W21x147
W33x130
W18x143
W18x158
W30x99
W27x114
W30x108
W21x122

AMPLE: Lb= 14 ft, Mu = 1150 kft, Cb = 1



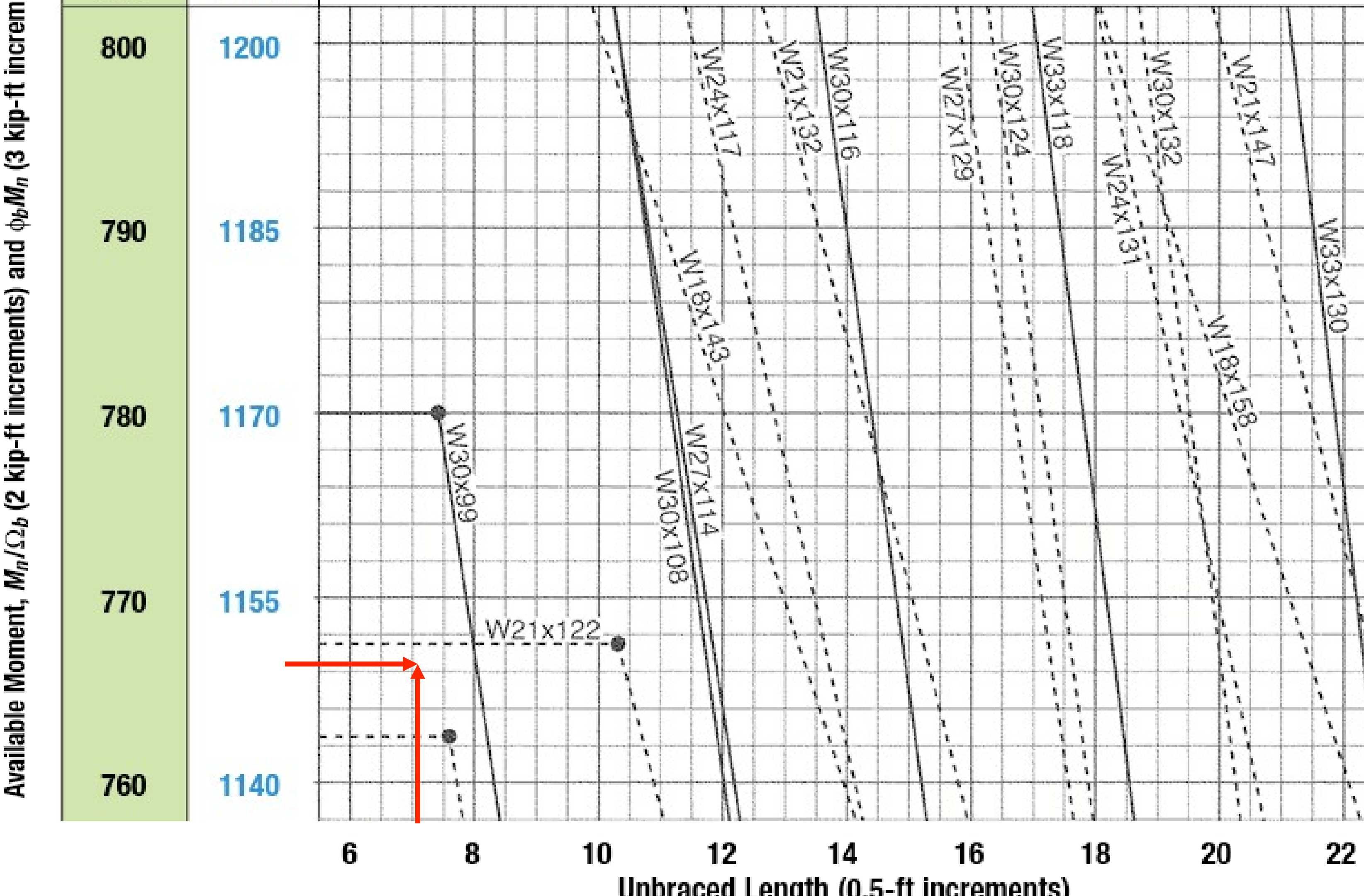
AMPLE: Lb= 7 ft, Mu = 1150 kft, Cb = 1.0

$F_y = 50$ ksi $C_b = 1$	
M_n/Ω_b	$\phi_b M_n$
kip-ft	kip-ft
ASD	LRFD

Table 3-10 (continued)

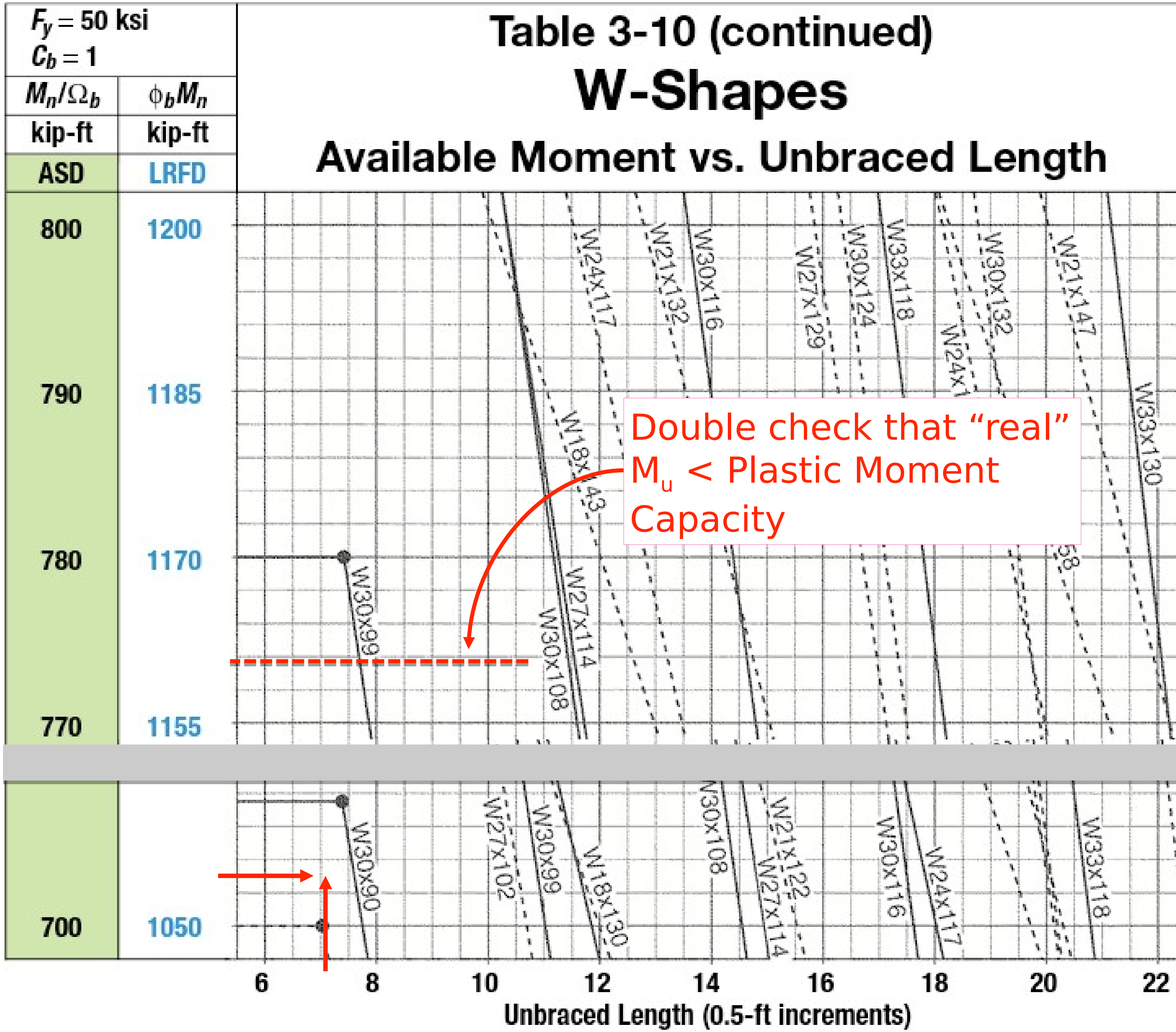
W-Shapes

Available Moment vs. Unbraced Length



MPLE: Lb= 7 ft, Mu = 1160 kft, Cb = 1.1

tional" Mu = Mu/Cb = 1160/1.1 = 1055 k

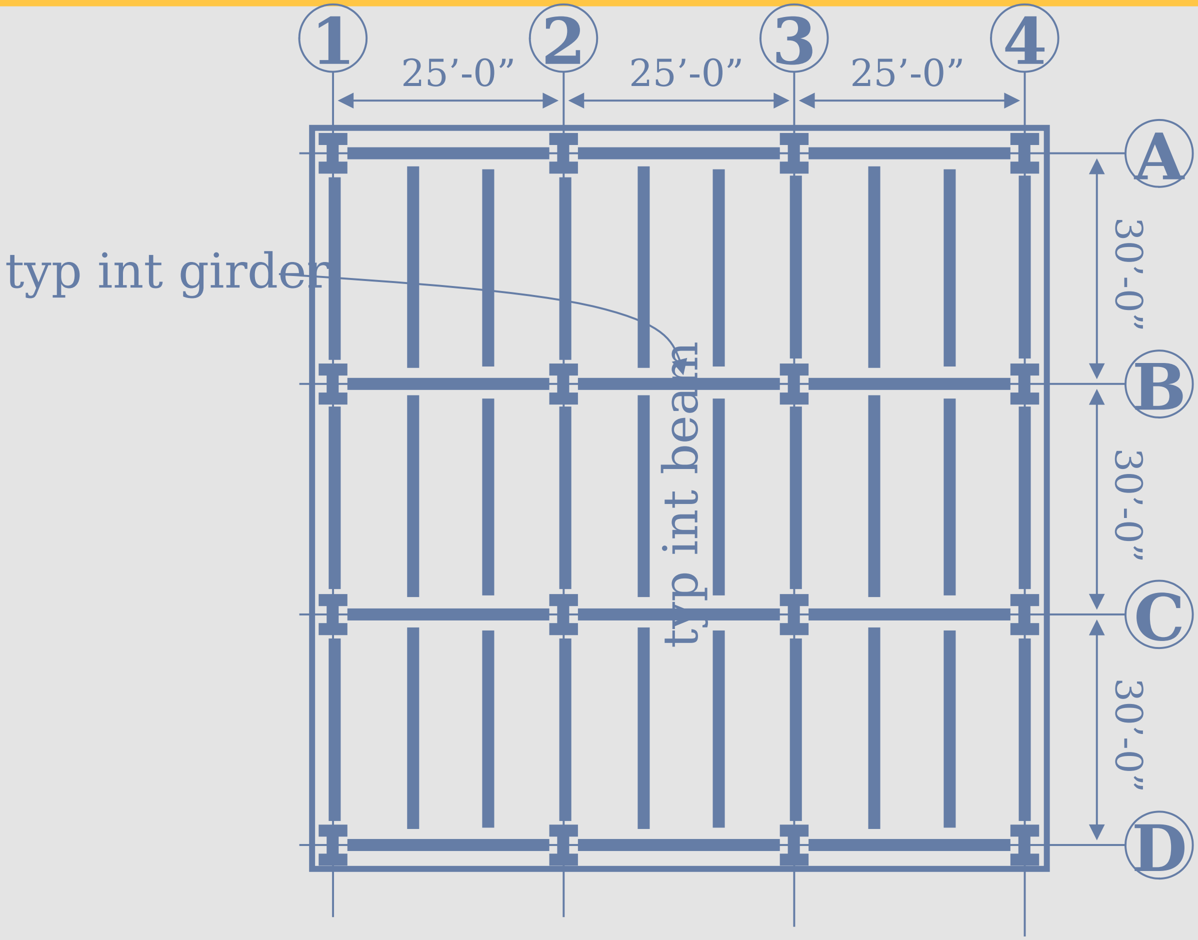


Questions?

Example Problem

- Design the most efficient typical interior WF beam.
- Design the most efficient typical interior WF girder.
- Assumptions:
  - Floor Plans are on the following page
  - Floor Loading: Dead = 100psf, Live = 40 psf (reducible)
  - Self weight of the members is included in the dead load
  - A992 Steel
  - Beam compression flanges are unbraced
  - Girder compression flanges are braced only at beam connection locations
  - Deflection Limits: Dead + Live = L/240, Live = L/360
  - Live Load Reductions: $L = L_0[0.25+15/\mathrm{sqrt}(K_{LL}\ A_T)]$, $L \geq 0.50L_0$ (ASCE 7-10 4.8)

Example Problem



Floor Framing Plan

Example Problem

- **Typical Interior Beam**

  - **Calculate Loads**

    DL = 100 psf; LL_0 = 40 psf

    Trib Area = (25'/3) (30') = 250 ft^2

    $LL = LL_0\ [0.25 + 15 / \mathrm{sqrt}(K_{LL}\ A_T)]$

    $= (40\mathrm{psf})\ [0.25 + 15 / \mathrm{sqrt}\ (2 \times 250\mathrm{ft}^2)]$

    = 37 psf

Example Problem

- Calculate Applied Loads
  - $w = DL + LL = (100psf + 37psf)(25'/3)$
    $= 1.14klf$
  - $w_L = LL = (37psf)(25'/3)$
    $= 0.31klf$
  - $w_u = 1.2DL + 1.6LL$
    $= [(1.2)(100psf) + (1.6)(37psf)](25'/3)$
    $= 1.49\ klf$
- Calculate Internal Forces

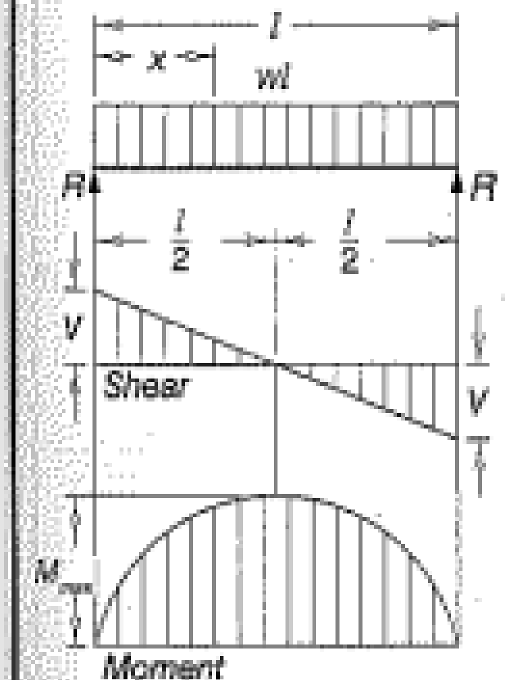
$$M_u = w_u L^2 / 8 = (1.49klf)(30')^2 / 8 = 168kft$$

$$V_u = w_u L / 2 = (1.49klf)(30') / 2 = 22.4k$$

Example Problem

- Table 3-23 (pg 3-211)

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) .... $= \frac{wl^2}{8}$

M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

Example Problem

- CALCULATE CAPACITY USING EQUATIONS
- Try W12x53
  - Check Flexural Capacity

    Top Flange UnBraced (L_b = 30')

    $_bM_n = C_b\ [_bM_p - BF(L_b\text{-}L_p)] \leq \phi_b M_p$

    where C_b = 1.14 (Table 3-1)

    = 1.14 [292 – 5.48(30-8.76)]

    = 200.1 k-ft < 292 k-ft

    $_bM_n$ = 200.1 > 168 O.K.
  - Check Shear Capacity

    $_vV_n$ = 125 k O.K.

F_y = 50 ksi

Table 3–2 (continued)
W Shapes
Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF	BF	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W21×55	**126**	**314**	**473**	**192**	**289**	**10.8**	**16.3**	**6.11**	**17.4**	**1140**	**156**	**234**
W14×74	126	314	473	196	294	5.34	8.03	8.76	31.0	795	128	191
W18×60	123	307	461	189	284	9.64	14.5	5.93	18.2	984	151	227
W12×79	119	297	446	187	281	3.77	5.67	10.8	39.9	662	116	175
W14×68	115	287	431	180	270	5.20	7.81	8.69	29.3	722	117	175
W10×88	113	282	424	172	259	2.63	3.95	9.29	51.1	534	131	197
W18×55	**112**	**279**	**420**	**172**	**258**	**9.26**	**13.9**	**5.90**	**17.5**	**890**	**141**	**212**
W21×50	**110**	**274**	**413**	**165**	**248**	**12.2**	**18.3**	**4.59**	**13.6**	**984**	**158**	**237**
W12×72	108	269	405	170	256	3.72	5.59	10.7	37.4	597	105	158
W21×48[f]	**107**	**265**	**398**	**162**	**244**	**9.78**	**14.7**	**6.09**	**16.6**	**959**	**144**	**217**
W16×57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212
W14×61	102	254	383	161	242	4.96	7.46	8.65	27.5	640	104	156
W18×50	101	252	379	155	233	8.69	13.1	5.83	17.0	800	128	192
W10×77	97.6	244	366	150	225	2.59	3.90	9.18	45.2	455	112	169
W12×65[f]	96.8	237	356	154	231	3.60	5.41	11.9	35.1	533	94.5	142
W21×44	**95.4**	**238**	**358**	**143**	**214**	**11.2**	**16.8**	**4.45**	**13.0**	**843**	**145**	**217**
W16×50	92.0	230	345	141	213	7.59	11.4	5.62	17.2	659	124	185
W18×46	90.7	226	340	138	207	9.71	14.6	4.56	13.7	712	130	195
W14×53	87.1	217	327	136	204	5.27	7.93	6.78	22.2	541	103	155
W12×58	86.4	216	324	136	205	3.76	5.66	8.87	29.9	475	87.8	132
W10×68	85.3	213	320	132	199	2.57	3.86	9.15	40.6	394	97.8	147
W16×45	82.3	205	309	127	191	7.16	10.8	5.55	16.5	586	111	167
W18×40	**78.4**	**196**	**294**	**119**	**180**	**8.86**	**13.3**	**4.49**	**13.1**	**612**	**113**	**169**
W14×48	78.4	196	294	123	184	5.10	7.66	6.75	21.1	484	93.8	141
W12×53	77.9	194	292	123	185	3.65	5.48	8.76	28.2	425	83.2	125
W10×60	74.6	186	280	116	175	2.53	3.80	9.08	36.6	341	85.8	129
W16×40	**73.0**	**182**	**274**	**113**	**170**	**6.69**	**10.1**	**5.55**	**15.9**	**518**	**97.7**	**146**
W12×50	71.9	179	270	112	169	3.97	5.97	6.92	23.9	391	90.2	135
W8×67	70.1	175	263	105	159	1.73	2.60	7.49	47.7	272	103	154
W14×43	69.6	174	261	109	164	4.82	7.24	6.68	20.0	428	83.3	125
W10×54	66.6	166	250	105	158	2.49	3.74	9.04	33.7	303	74.7	112

ASD	LRFD
$\Omega_b = 1.67$ $\Omega_v = 1.50$	$\phi_b = 0.90$ $\phi_v = 1.00$

[f] Shape exceeds compact limit for flexure with F_y = 50 ksi.

F_y = 50 ksi

Table 3–2 (continued)
W Shapes
Selection by Z_x

Z_x

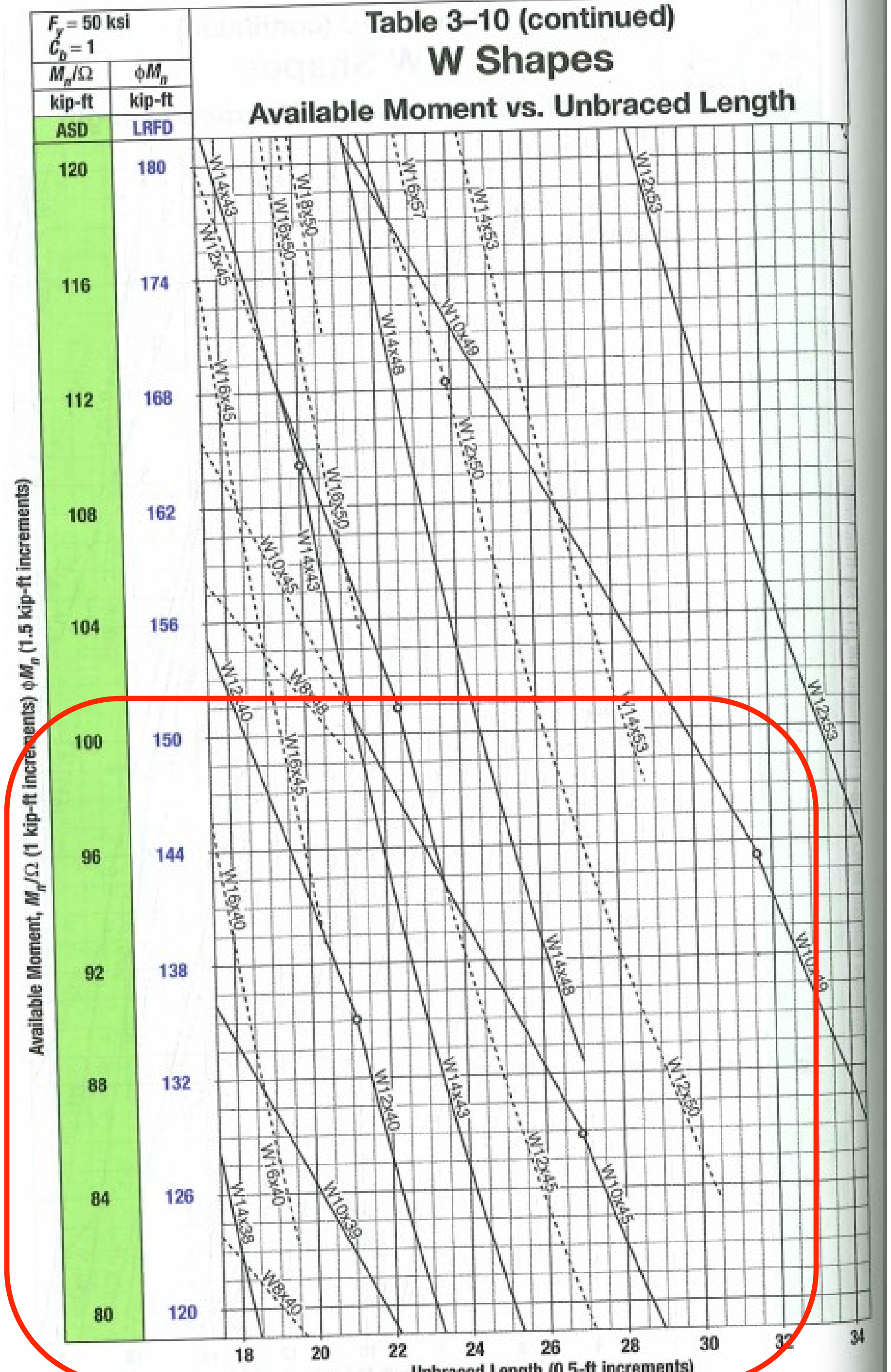
Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF		L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W18×40	78.4	196	294	119	180	8.86	13.3	4.49	13.1	612	113	169
W14×48	78.4	196	294	123	184	5.10	7.66	6.75	21.1	484	93.8	141
W12×53	77.9	194	292	123	185	3.65	5.48	8.76	28.2	425	83.2	125
W10×60	74.6	186	280	116	175	2.53	3.80	9.08	36.6	341	85.8	129

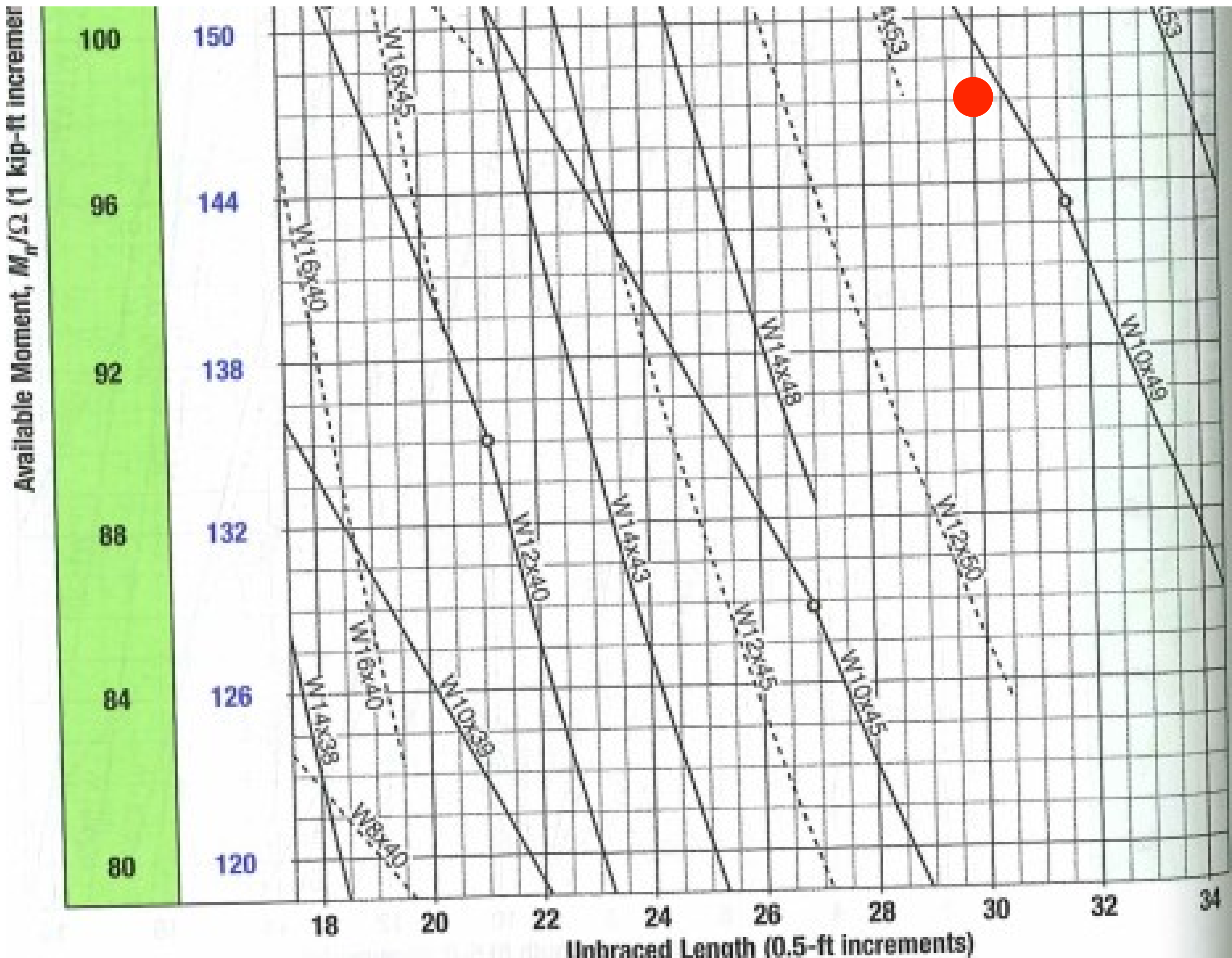
Example Problem

- DETERMINE CAPACITY USING CHARTS
- To read the charts based on $C_b = 1.0$
  - "Fictional" Demand for charts $= M_u / C_b$
    $= 168/1.14$
    $= 147$ k-ft

See next slide select W10x49

Table 3-10 (continued)
W Shapes
Available Moment vs. Unbraced Length





Available Moment, M_n/Ω (1 kip-ft increme
100
96
92
88
84
80
150
144
138
132
126
120
W16x45
W16x40
W14x48
W10x49
W12x40
W14x43
W12x50
W12x45
W10x45
W16x40
W10x39
W14x38
W8x40
18
20
22
24
26
28
30
32
34
Unbraced Length (0.5-ft increments)

Example Problem

- Check Serviceability
  - Live Load Deflection

Δ_{max} (at center) .... $= \dfrac{5wl^4}{384EI}$

$\Delta_L = 5\ w\ L^4 / 384\ E\ I$

$= (5)(0.31\text{klf})(30')^4(12''/\text{ft})^3/(384)(29\text{E}3\text{ksi})(272\text{in}^4)$

$= 0.72'' < L/360 = (30')(12''/\text{ft})/360 = 1''$

therefore OK

  - Dead Load + Live Load Deflection

$\Delta_{D+L} = 5\ w\ L^4 / 384\ E\ I$

$= (5)(1.14\text{klf})(30')^4(12''/\text{ft})^3/(384)(29\text{E}3\text{ksi})(272\text{in}^4)$

$= 2.63'' > L/240 = (30')(12''/\text{ft})/240 = 1.5''$

Table 3-3 (continued)
W Shapes
Selection by I_x

I_X

Shape	I_x in.^{4}
W30×90	**3610**
W12×305[h]	3550
W24×117	3540
W18×175	3450
W14×257	3400
W27×94	3270
W21×132	3220
W12×279[h]	3110
W24×104	3100
W18×158	3060
W14×233	3010
W24×103	3000
W21×122	2960
W27×84	**2850**
W18×143	2750
W12×252[h]	2720
W24×94	2700
W21×111	2670
W14×211	2660
W18×130	2460
W21×101	2420
W12×230[h]	2420
W14×193	2400
W24×84	**2370**
W18×119	2190
W14×176	2140
W12×210	2140
W24×76	**2100**
W21×93	2070
W18×106	1910
W14×159	1900
W12×190	1890
W24×68	**1830**
W21×83	1830
W18×97	1750
W14×145	1710
W12×170	1650
W21×73	1600
W24×62	**1550**
W18×86	1530
W14×132	1530
W16×100	1490
W21×68	1480
W12×152	1430
W14×120	1380
W24×55	**1350**
W21×62	1330
W18×76	1330
W16×89	1300
W14×109	1240
W12×136	1240
W21×57	1170
W18×71	1170
W21×55	**1140**
W16×77	1110
W14×99	1110
W18×65	1070
W12×120	1070
W14×90	999
W21×50	**984**
W18×60	984
W21×48	**959**
W16×67	954
W12×106	933
W18×55	890
W14×82	881
W21×44	**843**
W12×96	833
W18×50	800
W14×74	795
W16×57	758
W12×87	740
W14×68	722
W10×112	716
W18×46	712
W12×79	662
W16×50	659
W14×61	640
W10×100	623
W18×40	**612**
W12×72	597
W16×45	586
W14×53	541
W10×88	534
W12×65	533
W16×40	**518**
W18×35	**510**
W14×48	484
W12×58	475
W10×77	455
W16×36	448
W14×43	428
W12×53	425
W10×68	394
W12×50	391
W14×38	385
W16×31	**375**
W12×45	348
W10×60	341
W14×34	340
W12×40	307
W10×54	303
W16×26	**301**
W14×30	291
W12×35	285
W10×49	272
W8×67	272
W10×45	248
W14×26	**245**
W12×30	238
W8×58	228
W10×39	209
W12×26	**204**
W14×22	**199**
W8×48	184
W10×33	171
W10×30	170
W12×22	**156**
W8×40	146
W10×26	144
W12×19	**130**
W8×35	127
W10×22	118
W8×31	110
W12×16	**103**
W8×28	98.0
W10×19	96.3
W12×14	**88.6**
W8×24	82.7
W10×17	81.9
W8×21	75.3
W10×15	68.9
W8×18	61.9
W10×12	**53.8**
W8×15	48.0
W8×13	39.6
W8×10	**30.8**

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.

Shape	I_x in.^{4}
W16×26	**301**
W14×30	291
W12×35	285
W10×49	272
W8×67	272
W10×45	248

Example Problem

- Check Serviceability
  - Determine I_{reqd}

    I_{reqd} = [(2.63”)/(1.5”)] (272 in^4) = 477in^4

    Options:

    W18x35 (I = 510; Z = 66.5) *select lightest*

    W16x40 (I = 518; Z = 72.9)

Use W18x35 typical interior beam

Example Problem

- **<u>Typical Interior Girder</u>**
  - **Calculate Loads**

    DL = 100 psf; LL_0 = 40 psf

    Trib Area = (2/3) (25') (30') = 500 ft^2

    $LL = LL_0 [0.25 + 15 / \mathrm{sqrt}(K_{LL} A_T)]$

    = (40psf) [0.25 + 15 / sqrt (2 x 500ft^2)]

    = 29 psf

Example Problem

- Calculate Applied Loads
  - $P_D = (100psf)\ (25'/3)\ (30') = 25.0k$
  - $P_L = (29psf)\ (25'/3)\ (30') = 7.3k$
  - $P_s = P_D + P_L = 25.0k + 7.3k = 32.3k$
  - $P_u = 1.2P_D + 1.6P_L$

    $= (1.2)(25.0k) + (1.6)(7.3k) = 41.7k$
- Calculate Internal Forces

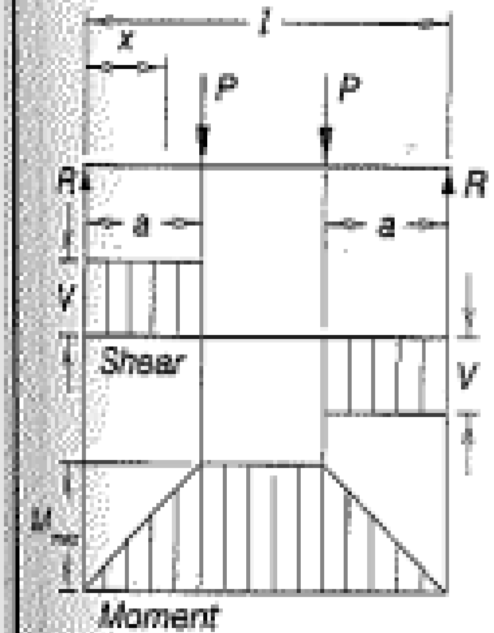
  $M_u = P_u\ L / 3 = (41.7k)\ (25'/3) / 3 =$ 347k-ft

  $V_u = P_u = 41.7k$

Example Problem

- Table 3-23 (pg 3-213)

9. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load .... $= \frac{8Pa}{l}$

$R = V$ .... $= P$

M_{max} (between loads) .... $= Pa$

M_x (when $x < a$) .... $= Px$

Δ_{max} (at center) .... $= \frac{Pa}{24EI}\left(3l^2 - 4a^2\right)$

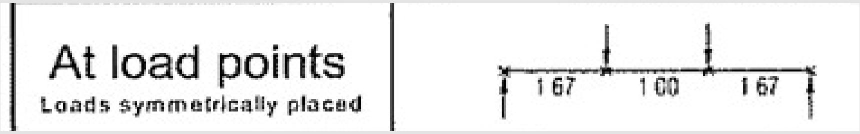
Δ_{max} (when $a = \frac{l}{3}$) .... $= \frac{Pl^3}{28EI}$

Δ_x (when $x < a$) .... $= \frac{Px}{6EI}\left(3la - 3a^2 - x^2\right)$

Δ_x (when $a < x < (l - a)$) .... $= \frac{Pa}{6EI}\left(3lx - 3x^2 - a^2\right)$

Example Problem

- CALCULATE CAPACITY USING EQUATIONS
- Try W24x62
  - Check Flexural Capacity



Top Flange UnBraced (L_b = 8.33’)

$\varphi_b M_n = C_b\,[\varphi_b M_p - BF(L_b - L_p)] \le \varphi_b M_p$

where C_b = 1.00 (Table 3-1)

= 1.00 [574 – 24.1(8.33-4.87)]

= 490 k-ft < 574 k-ft

M_u = 347 < 490 O.K.

  - Check Shear Capacity

$\varphi_v V_n$ = 306 k *from tables* > V_u = 41.7k

Z_x

Table 3–2 (continued)
W Shapes
Selection by Z_x

F_y = 50 ksi

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF	BF	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W24×84	**224**	**559**	**840**	**342**	**515**	**16.2**	**24.3**	**6.89**	**20.3**	**2370**	**227**	**340**
W21×93	221	551	829	335	504	14.6	21.9	6.50	21.3	2070	251	376
W12×136	214	534	803	325	488	4.01	6.03	11.2	63.3	1240	212	318
W14×120	212	529	795	332	499	5.09	7.64	13.2	52.0	1380	171	256
W18×97	211	526	791	328	494	9.45	14.2	9.36	30.3	1750	199	298
W24×76	**200**	**499**	**750**	**307**	**462**	**15.0**	**22.5**	**6.78**	**19.6**	**2100**	**210**	**316**
W16×100	198	494	743	306	459	7.90	11.9	8.87	32.7	1490	199	298
W21×83	196	489	735	299	449	13.8	20.8	6.46	20.2	1830	221	331
W14×109	192	479	720	302	454	5.02	7.54	13.2	48.4	1240	150	226
W18×86	186	464	698	290	436	9.04	13.6	9.29	28.5	1070	177	265
W12×120	186	464	698	285	428	3.95	5.93	11.1	56.5	1530	186	279
W24×68	**177**	**442**	**664**	**269**	**404**	**14.1**	**21.2**	**6.61**	**18.8**	**1830**	**197**	**295**
W16×89	175	437	656	271	407	7.74	11.6	8.80	30.2	1300	176	264
W14×99^f	173	430	646	274	412	4.89	7.35	13.5	45.3	1110	137	206
W21×73	172	429	645	264	396	12.9	19.4	6.39	19.2	1600	193	290
W12×106	164	409	615	253	381	3.93	5.90	11.0	50.7	933	157	236
W18×76	163	407	611	255	383	8.49	12.8	9.22	27.1	1330	155	232
W21×68	**160**	**399**	**600**	**245**	**368**	**12.5**	**18.8**	**6.36**	**18.7**	**1480**	**182**	**273**
W14×90^f	157	382	573	250	375	4.80	7.22	15.2	42.6	999	123	185
W24x62	**153**	**382**	**574**	**229**	**344**	**16.0**	**24.1**	**4.87**	**14.4**	**1550**	**204**	**306**
W16×77	150	374	563	234	352	7.34	11.0	8.72	27.8	1110	150	225
W12×96	147	367	551	229	344	3.87	5.81	10.9	46.6	833	140	210
W10×112	147	367	551	220	331	2.68	4.02	9.47	64.3	716	172	257
W18×71	146	364	548	222	333	10.5	15.7	6.00	19.6	1170	183	274
W21×62	**144**	**359**	**540**	**222**	**333**	**11.6**	**17.4**	**6.25**	**18.1**	**1330**	**168**	**252**
W14×82	139	347	521	215	323	5.43	8.16	8.76	33.1	881	146	219
W24×55^v	**134**	**334**	**503**	**199**	**299**	**14.8**	**22.2**	**4.73**	**13.9**	**1350**	**167**	**251**
W18×65	133	332	499	204	307	9.92	14.9	5.97	18.8	1070	165	248
W12×87	132	329	495	206	310	3.84	5.76	10.8	43.0	740	129	194
W16×67	130	324	488	204	307	6.91	10.4	8.69	26.1	954	129	194
W10×100	130	324	488	196	294	2.66	4.01	9.36	57.7	623	151	226
W21×57	129	322	484	194	291	13.4	20.1	4.77	14.3	1170	171	256

ASD	LRFD
$\Omega_b = 1.67$ $\Omega_v = 1.50$	$\phi_b = 0.90$ $\phi_v = 1.00$

^{f} Shape exceeds compact limit for flexure with F_y = 50 ksi.

^{v} Shape does not meet the h/t_w limit for shear in Specification Section G2.1a with F_y = 50 ksi, $\Omega_v = 1.67$, $\phi_v = 0.90$.

Z_X

Table 3-2 (continued)
W Shapes
Selection by Z_x

F_y = 50 ksi

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF		L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W24x62	**153**	**382**	**574**	**229**	**344**	**16.0**	**24.1**	**4.87**	**14.4**	**1550**	**204**	**306**
W16×77	150	374	563	234	352	7.34	11.0	8.72	27.8	1110	150	225
W12×96	147	367	551	229	344	3.87	5.81	10.9	46.6	833	140	210
W10×112	147	367	551	220	331	2.68	4.02	9.47	64.3	716	172	257
W18×71	146	364	548	222	333	10.5	15.7	6.00	19.6	1170	183	274

Example Problem

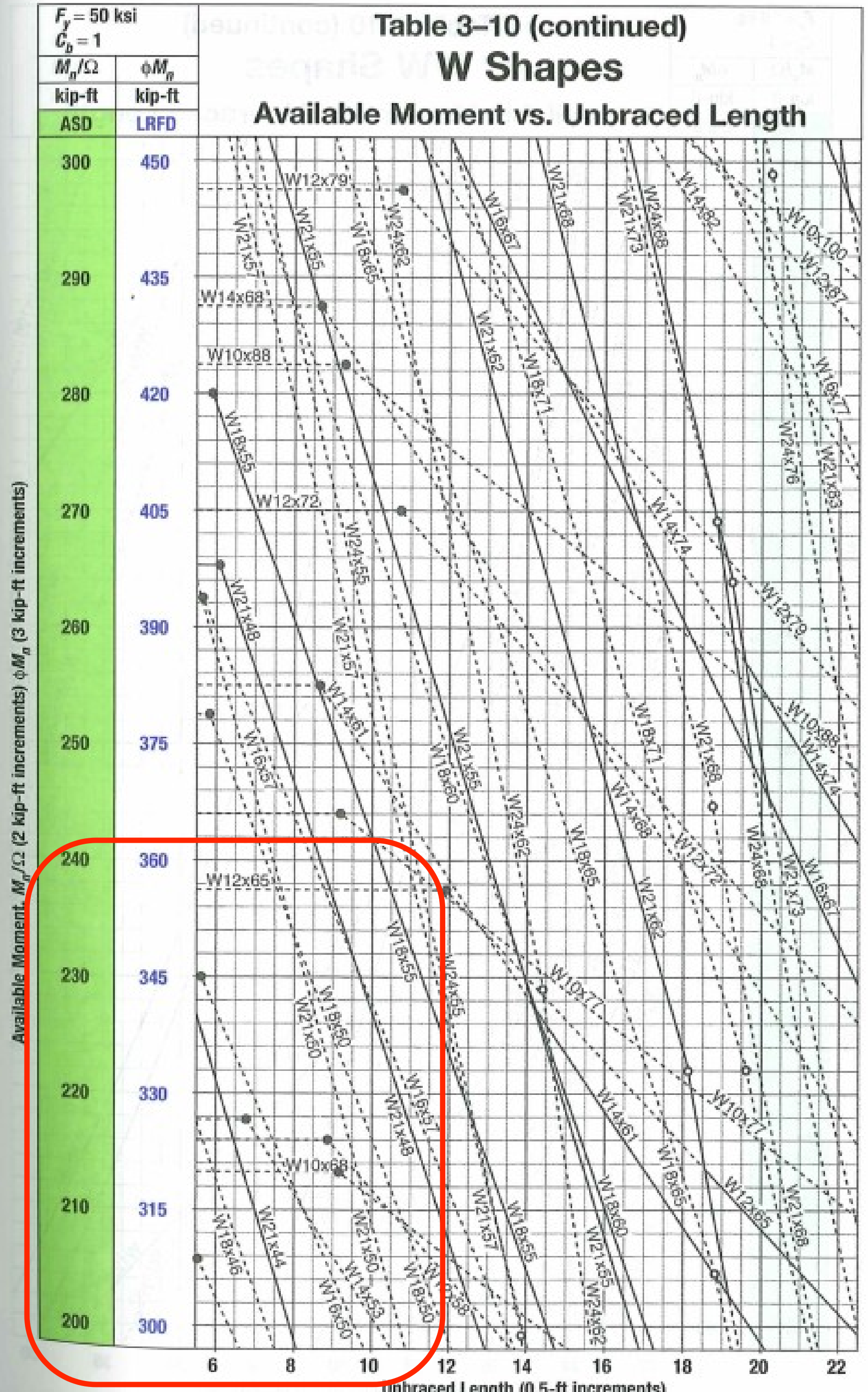
- DETERMINE CAPACITY USING CHARTS
- To read the charts based on $C_b = 1.0$
  - "Fictional" Demand for charts $= M_u / C_b$

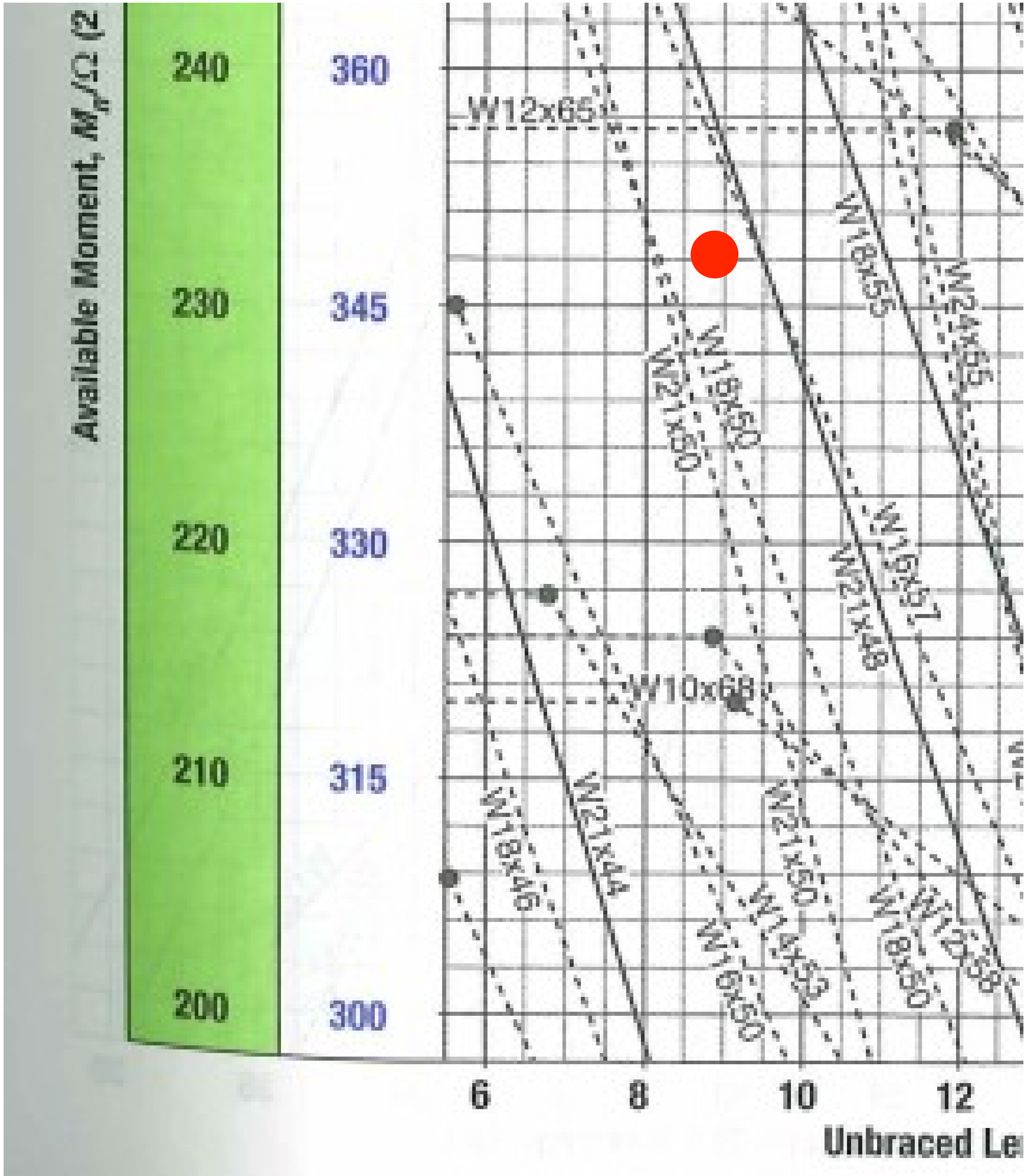
    $= 347/1.00$

    $= 347$ k-ft

See next slide select W21x48

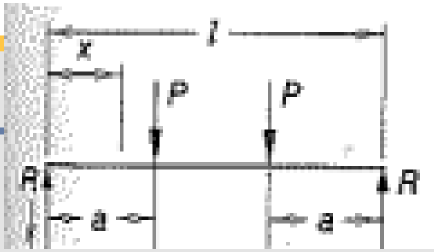
Table 3–10 (continued)
W Shapes
Available Moment vs. Unbraced Length





Available Moment, M_n/Ω (2
240
230
220
210
200
360
345
330
315
300
W12x65
W18x55
W24x55
W16x60
W21x50
W16x57
W21x48
W10x68
W18x46
W21x44
W21x50
W14x53
W16x50
W18x50
W12x58
6
8
10
12
Unbraced Le

Example Problem



- Check Serviceability (W21x4
  - Live Load Deflection

Δ_{max} (at center) .... $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δ_L = (P a / 24 E I) (3l^2 – 4a

= [(7.3k)(25'/3)(12"/ft)^3/(24)(29E3ksi)(959in^4)] [3(25')^2- 4(25'/3)^2]

= 0.26" < L/360 = (25')(12"/ft)/360 = 0.83" *so OK*

  - Dead Load + Live Load Deflection

W21×48	959
W16×67	954
W12×106	933
W18×55	890
W14×82	881

= *ratio the deflections*

= (32.2k/7.3k) (0.26")

= 1.14" < L/240 = (25')(12"/ft)/240 = 1.25" *so OK*

Use W21x48 typical interior girder